

The internal structures of the nucleon resonances $N(1875)$ and $N(2120)$

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A nucleon resonance with spin-parity $J^P = 3/2^-$ and mass about 2.1 GeV is essential to reproduce the photoproduction cross sections for $\Lambda(1520)$ released by the LEPS and CLAS Collaborations. It can be explained as the third nucleon resonance state $[3/2^-]_3$ in the constituent quark model so that there is no position to settle the $N(1875)$ which is listed in the PDG as the third $N3/2^-$ nucleon resonance. An interpretation is proposed that the $N(1875)$ is from the interaction of a decuplet baryon $\Sigma(1385)$ and a octet meson K , which is favored by a calculation of binding energy and decay pattern in a Bethe-Salpeter approach.

PACS numbers: 14.20.Gk, 11.10.St, 14.20.Pt

I. INTRODUCTION

In new versions of the Review of Particle physics (PDG) after the year 2012 [1], there are four $N3/2^-$ states, $N(1520)$, $N(1700)$, $N(1875)$ and $N(2120)$. The two-star state $N(2080)$ in previous versions has been split into a three-star $N(1875)$ and a two-star $N(2120)$ based on the evidence from BnGa analysis [2].

Usually the $N(1520)$ and the $N(1700)$ are assigned to states with orbital angular momentum $L = 1$ in quark model, and mixing effect is very important to explain the decay pattern of these states [3]. The situation for the internal structures of two $N3/2^-$ states with higher mass, the $N(1875)$ and the $N(2120)$, is much less unclear. In quark model, the $N(1875)$ and the $N(2120)$ are in the mass region of $N = 3$ band states of which the masses and decay patterns were predicted [4, 5]. However, the explicit correspondence between predicted and observed states is unclear. In Large N_c QCD, the third and fourth $N3/2^-$ states have masses 2101 ± 14 and 2170 ± 42 MeV, respectively [6]. Klempt and others claimed that the $N(1875)$ is the missing third $N(3/2^-)$ state in mass region 1800 – 1900 MeV with orbit angular momentum $L = 1$ and radial excitation number $N = 1$ [7], which is also supported by the AdS/QCD [8]. Their conclusion is only based on a comparison between predicted and observed masses. As enlightened by Isgur, “in a complex system like the baryon resonances, predicting the spectrum of states is not a very stringent test of a model” [9]. Decay pattern provides more information about hadron internal structure.

Many analyses suggested that a $N3/2^-$ state with mass about 2.1 GeV is essential to explain experimental results [10–13]. Before the year 2012, it is related to the only $N3/2^-$ state listed in the PDG with mass higher than 1.8 GeV, the $N(2080)$, and explained as the third state $[N3/2^-]_3$ predicted in the constituent quark model. For example, the $N(2080)$ is found to play the most important role in the photoproduction of $\Lambda(1520)$ off proton target [12, 13]. Recently, the CLAS Collaboration at Jefferson National Accelerator Facility re-

leased their exclusive photoproduction cross section for the $\Lambda(1520)$ for energies from near threshold up to a center of mass energy W of 2.85 GeV with large range of the K production angle [14]. The reanalyses about the new data in Refs. [15, 19] confirmed the previous conclusion that a nucleon resonance near 2.1 GeV, $N(2120)$, is essential to reproduce the experimental data [12, 13].

II. ROLE OF THE $N(2120)$ IN THE $\Lambda(1520)$ PHOTOPRODUCTION

In the following it will be shown why the $N(2120)$ should be assigned to the third state $[N3/2^-]_3$ in the constituent quark model in line with the theoretical framework in Ref. [15]. There are five $N3/2^-$ states in $N = 3$ band, of which the radiative and $\Lambda(1520)K$ decay amplitudes were predicted in Refs. [4, 5] as listed in Table. I.

TABLE I: The $N3/2^-$ nucleon resonances and their decay amplitudes predicted in the relativistic quark model [4, 5]. The mass m_R , helicity amplitudes $A_{1/2,3/2}$ and partial wave decay amplitudes $G(\ell)$ are in the unit of MeV, $10^{-3}/\sqrt{\text{GeV}}$ and $\sqrt{\text{MeV}}$, respectively.

State	m_R	$A_{1/2}^p$	$A_{3/2}^p$	$G(\ell_1)$	$G(\ell_2)$
$[N3/2^-]_3$	1960	36	-43	-2.6	-0.2
$[N3/2^-]_4$	2055	16	0	-0.5	0.0
$[N3/2^-]_5$	2095	-9	-14	0.4	0.0
$[N3/2^-]_6$	2165	--	--	0.4	0.0
$[N3/2^-]_7$	2180	--	--	1.1	0.1

The predicted radiative and strong decay amplitudes suggest the importance of the nucleon resonance $[N3/2^-]_3$ in the $\Lambda(1520)$ photoproduction, which is the first state in $N = 3$ band states and the third state in all nucleon resonances with $J^P = 3/2^-$ predicted in the constituent quark model.

In Ref. [15], based on the high precision experimental data released by the CLAS and LEPS Collaborations recently, the interaction mechanism of the photoproduction of $\Lambda(1520)$ off a proton target is investigated within a Regge-plus-resonance approach. The inclusion of the $N(2120)$ as state $[N3/2^-]_3$ in the constituent quark model reduced the χ^2 obviously. In that

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work, mass and width are fixed at 2.12 GeV and 0.33 GeV, respectively. Here, a mass scan is made for the $N(2120)$ by fitting the data from the CLAS and LEPS Collaborations. Except mass, the width Γ_R , which was fixed at 0.33 GeV in previous work [15], is also set as a free parameter. The behavior of χ^2 is presented in Fig. 1.

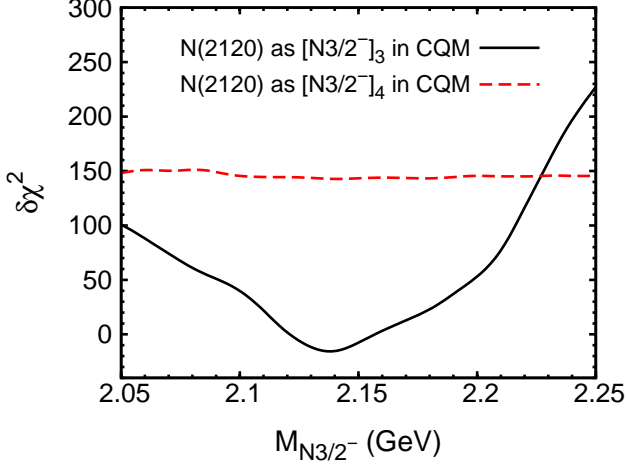


FIG. 1: (Color online) The change of χ^2 in mass scan. The solid and dashed lines are for the results with assuming the $N(2120)$ as state $[N3/2^-]_3$ and with assuming the $N(2120)$ as state $[N3/2^-]_4$, respectively.

Here the results with assuming the $N(2120)$ as state $[N3/2^-]_3$ and with assuming the $N(2120)$ as $[N3/2^-]_4$ are provided. If the $N(2120)$ is assumed to be the third state $[N3/2^-]_3$ in the constituent quark model, the change of χ^2 will decrease and reach minimum at 2.13 GeV with the increase of mass. If assumed to be the fourth state $[N3/2^-]_4$, the change of χ^2 keep stable around 150, which means that the experimental data can not be well reproduced. Obviously, the $N(2120)$ should be assigned as state $[N3/2^-]_3$ instead of state $[N3/2^-]_4$ in the constituent quark model. Since the $N(1875)$ is much lower than the $N(2120)$, it is unnatural to assign it to the fourth or higher states. The first and second states in the constituent quark model have been assigned to the four-star $N(1520)$ and the three-star $N(1700)$ in the PDG, which has been confirmed by many experimental and theoretical evidences [1]. Hence, there is no position to settle the $N(1875)$ in the constituent quark model.

The state $[N3/2^-]_3$ predicted in the constituent quark model is much lower than the $N(2120)$ even if model uncertainty, about 100 MeV, is considered. It is well-known that loop effect will lead to mass shift. The state $[N3/2^-]_3$ has a large decay width in $\Lambda(1520)K$ channel as predicted in the constituent quark model [4, 5]. Moreover, the $\Lambda(1520)K$ threshold is near bare mass of state $[N3/2^-]_3$. Hence, the large difference between bare mass and observed mass can be explained by both uncertainty of the constituent quark model and mass shift arising from the $\Lambda(1520)K$ loop effect.

III. THE $N(1875)$ AS A $\Sigma(1385)K$ BOUND STATE

Now that the $N(1875)$ can not be explained in the conventional quark model, it may be a exotic hadron. In meson sector, some of particles which can not be explained in the quark model framework, such as XYZ particles observed in recent years, have been suggested to be hadronic molecular states. The light scalars $a_0(980)$, $f_0(980)$ and $f_0(500)$ are often considered as meson-meson resonances. In baryon sector, some authors proposed that the $\Lambda(1405)$ may be explained as a $N\bar{K}$ bound state [16–18]. The mass of the $N(1875)$ is close to the $\Sigma(1385)K$ threshold, which encourages us to interpret $N(1875)$ as a bound state of Σ^* and K (here and hereafter I denote $\Sigma(1385)$ as Σ^*). As said above, the internal structure of a hadron can not be judged only through its mass. In this work, both mass and decay pattern of $N(1875)$ as a bound state of Σ^* and K will be calculated with a method developed based on the covariant spectator formalism of the Bethe-Salpeter equation [20–25], which has been used to study the $B\bar{B}^*$ and the $D\bar{D}^*$ systems.

Analogous to Ref. [24, 25], with help of onshellness of the heavy constituent 1, Σ^* , the numerator of propagator $P_1^{\mu\nu}$ is rewritten as $\sum_{\lambda} u_{1\lambda}^{\mu} \bar{u}_{1\lambda}^{\nu}$ with $u_{1\lambda}^{\mu}$ being the Rarita-Schwinger spinor with helicity λ . The equation for vertex is in a form

$$|\Gamma_{\lambda}\rangle = \sum_{\lambda'} \mathcal{V}_{\lambda\lambda'} G_0 |\Gamma_{\lambda'}\rangle, \quad (1)$$

with $|\Gamma_{\lambda}\rangle = \bar{u}_{\lambda}^{\mu} |\Gamma_{\mu\nu}\rangle u_{\lambda}^{\nu}$ and $\mathcal{V}_{\lambda\lambda'} = \bar{u}_{\lambda}^{\mu} \mathcal{V}_{\mu\nu} u_{\lambda'}^{\nu}$. The rest of propagator G_0 for particle 1 and 2 with mass m_1 and m_2 written down in the center of mass frame where $P = (W, \mathbf{0})$ is

$$G_0 = 2\pi i \frac{\delta^+(k_1^2 - m_1^2)}{k_2^2 - m_2^2} = 2\pi i \frac{\delta^+(k_1^0 - E_1(\mathbf{k}))}{2E_1(\mathbf{k})[(W - E_1(\mathbf{k})^2 - E_2^2(\mathbf{k}))]}, \quad (2)$$

where $k_1 = (k_1^0, \mathbf{k}) = (E_1(\mathbf{k}), \mathbf{k})$, $k_2 = (k_2^0, -\mathbf{k}) = (W - E_1(\mathbf{k}), -\mathbf{k})$ with $E_{1,2}(\mathbf{k}) = \sqrt{m_{1,2}^2 + |\mathbf{k}|^2}$.

The integral equation can be written explicitly as

$$(W - E_1(\mathbf{k}) - E_2(\mathbf{k}))\phi_{\lambda}(\mathbf{k}) = \sum_{\lambda'} \int \frac{d\mathbf{k}'}{(2\pi)^3} V_{\lambda\lambda'}(\mathbf{k}, \mathbf{k}', W) \phi_{\lambda'}(\mathbf{k}'), \quad (3)$$

with

$$V_{\lambda\lambda'}(\mathbf{k}, \mathbf{k}', W) = \frac{i \bar{\mathcal{V}}_{\lambda\lambda'}(\mathbf{k}, \mathbf{k}', W)}{\sqrt{2E_1(\mathbf{k})2E_2(\mathbf{k})2E_1'(\mathbf{k}')2E_2'(\mathbf{k}')}}, \quad (4)$$

where the reduced potential kernel $\bar{\mathcal{V}}_{\lambda\lambda'} = F(\mathbf{k})\mathcal{V}_{\lambda\lambda'}F(\mathbf{k}')$ with a factor as $F(\mathbf{k}) = \sqrt{2E_2(\mathbf{k})/(W - E_1(\mathbf{k}) + E_2(\mathbf{k}))}$. The normalized wave function can be related to vertex as $|\phi\rangle = N|\psi\rangle = NF^{-1}G_0|\Gamma_{\lambda}\rangle$ with the normalization factor $N(\mathbf{k}) = \sqrt{2E_1(\mathbf{k})E_2(\mathbf{k})/(2\pi)^5 W}$.

Since K is a pseudoscalar particle, it is forbidden to exchange pseudoscalar meson between K and Σ^* . The vector meson exchanges, ρ , ω and ϕ , is dominant in the interaction.

The potential kernel \mathcal{V} can be obtained from the effective Lagrangians describing the interactions for vector mesons V with K and Σ^* ,

$$\mathcal{L}_{KKV} = ig_{KKV} K^\dagger V^\mu \partial_\mu K, \quad (5)$$

$$\mathcal{L}_{\Sigma^* \Sigma^* V} = g_{\Sigma^* \Sigma^* V} \Sigma_\mu^{*\dagger} [\gamma^\nu - \frac{K_{\Sigma^* \Sigma^* V}}{2m_\Sigma} \sigma^{\mu\nu} \partial_\rho] V^\nu \Sigma_\mu. \quad (6)$$

In this work the isospin structures are following the standard form in Ref. [26] and omitted in the Lagrangians. The coupling constants for vector mesons ρ , ω and ϕ interacted with K and Σ^* can be obtained from $g_{\rho\pi\pi} = 6.199$ and $f_{\rho\Delta\Delta} = -4.30$ in quark model [27] and relations $g_{KK\rho} = g_{\rho\pi\pi}/2 = g_{KK\omega} = \sqrt{2}g_{KK\phi} = g_{\rho\pi\pi}/2$ and $g_{\Sigma^* \Sigma^* \rho} = -g_{\Sigma^* \Sigma^* \omega} = g_{\Sigma^* \Sigma^* \phi}/\sqrt{2} = g_{\Delta\Delta\rho}$ under $SU(3)$ symmetry. Here different definitions between Ref. [27] and this work have been considered. Since the constituent 2 is off shell, a monopole form factor is introduced at the vertex for each off-shell kaon meson with mass m_K as $h(k^2) = \Lambda^4/[(m_K^2 - k^2)^2 + \Lambda^4]$. The form factor for the exchanged meson with mass m_V is chosen as $f(q^2) = (\Lambda^2 - m_V^2)/(\Lambda^2 - q^2)$. Empirically the cut off Λ should be not far from 1 GeV.

The 3-dimensional equation can be reduced to a one-dimensional equation with partial wave expansion. The wave function has an angular dependent as

$$\phi_\lambda(\mathbf{k}) = \sqrt{\frac{2J+1}{4\pi}} D_{\lambda_R, \lambda}^{J*}(\phi, \theta, 0) \phi_{\lambda, \lambda_R}(|\mathbf{k}|), \quad (7)$$

where $D_{\lambda_R, \lambda}^{J*}(\phi, \theta, 0)$ is the rotation matrix with λ_R being the helicity of bound state with angular momentum J . The potential after partial wave expansion is

$$V_{\lambda\lambda'}^J(|\mathbf{k}|, |\mathbf{k}'|) = 2\pi \int d\cos\theta_{k,k'} d_{\lambda, \lambda'}^J(\theta_{k,k'}) V_{\lambda\lambda'}(\mathbf{k}, \mathbf{k}'), \quad (8)$$

where $\theta_{k,k'}$ is angle between \mathbf{k} and \mathbf{k}' . The one-dimensional integral equation reads

$$\begin{aligned} & (W - E_1(|\mathbf{k}|) - E_2(|\mathbf{k}|)) \phi_\lambda^J(|\mathbf{k}|) \\ &= \sum_{\lambda'} \int \frac{|\mathbf{k}'|^2 d|\mathbf{k}'|}{(2\pi)^3} V_{\lambda\lambda'}^J(|\mathbf{k}'|, |\mathbf{k}'|) \phi_{\lambda'}^J(|\mathbf{k}'|). \end{aligned} \quad (9)$$

To study the decay property of a bound state, the information about coupling of a bound state to its constituents is essential. In literatures it is often achieved with the method proposed by Weinberg [28, 29]. In this work, the vertex wave function, which contains the information about coupling of bound state to its constituents, is obtained during solving the binding energy. It make a study of the decay pattern of the $\Sigma^* K$ bound state possible.

Since two-body decay of a molecular state occurs only through hadron loop mechanism, it is suggested that three-body decay may be larger than two-body decay [30]. As shown in Refs. [30–32], it was found that three-body decay has positive correlation to the decay width of the constituents. Hence, the three-body decay of the bound state $\Sigma^* K$ is suppressed due to the small decay width of Σ^* . In this work the two-body decays through exchanging a particle between two

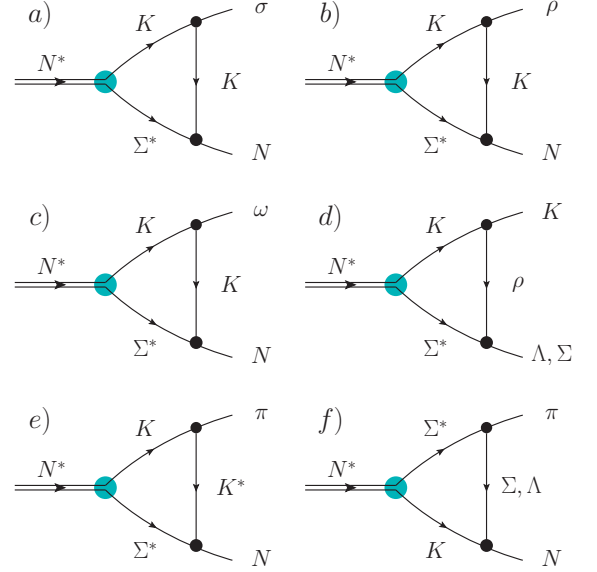


FIG. 2: (Color online) The decays of the $\Sigma^* K$ bound state. a) $N\sigma$ channel with K exchange. b) $N\rho$ channel with K exchange. c) $N\omega$ channel with K exchange. d) $K\Lambda/\Sigma$ channel with ρ exchange. e) and f) are for $N\pi$ decay channel with K^* exchange and with Λ/Σ exchange, respectively.

constituents as shown in Fig. 2 are taken as the main decay channels of the $\Sigma^* K$ bound state.

The decay amplitudes can be written as

$$\begin{aligned} \mathcal{M} &= \sum_{\lambda} A_{\lambda} G_0 |\Gamma_{\lambda}\rangle = \sum_{\lambda} A_{\lambda} F N^{-1} |\phi_{\lambda}\rangle \\ &\equiv \sum_{\lambda} \int \frac{d^4 k \delta(k_1^0 - E_1) F(|\mathbf{k}|)}{(2\pi)^4} \phi_{\lambda}(\mathbf{k}) A_{\lambda, \lambda'_1 \lambda'_2}(\mathbf{k}, \mathbf{k}'), \end{aligned} \quad (10)$$

where $\lambda'_{1,2}$ are helicities for two final particles and \mathbf{k} and \mathbf{k}' are the momenta for Σ^* and final meson in the center of mass frame. $A_{\lambda, \lambda'_1 \lambda'_2}$ is the amplitudes for two constituents Σ^* and K to two final particles, $N\pi$, $N\sigma$ and so on. The definitions of wave function ϕ and G_0 have been used in the derivation of Eq. (10). The normalization of wave function ϕ insures that there is no free total factor in our calculation of amplitude.

Besides the Lagrangians in Eq. (4), the following Lagrangians are used to calculate the amplitudes $A_{\lambda, \lambda'_1 \lambda'_2}$,

$$\mathcal{L}_{KK\sigma} = g_{KK\sigma} 2m_{\pi} \partial_{\mu} K^{\dagger} \partial^{\mu} K \sigma, \quad (11)$$

$$\mathcal{L}_{K^* K \pi} = ig_{K^* K \pi} K^{*\mu} (\pi \partial_{\mu} K - \partial_{\mu} \pi K), \quad (12)$$

$$\mathcal{L}_{KNY} = \frac{f_{KNY}}{m_N + m_Y} \bar{N} \gamma^{\mu} \gamma_5 Y \partial_{\mu} K + H.c., \quad (13)$$

$$\mathcal{L}_{PB\Sigma^*} = \frac{f_{PB\Sigma^*}}{m_P} \partial_{\mu} K \Sigma^{*\mu} N + H.c., \quad (14)$$

$$\mathcal{L}_{VB\Sigma^*} = -i \frac{f_{VB\Sigma^*}}{m_V} \Sigma^{*\mu} \gamma^{\nu} \gamma_5 [\partial_{\mu} \rho_{\nu} - \partial_{\nu} \rho_{\mu}] B + H.c., \quad (15)$$

where PB means KN , $\pi\Lambda$ or $\pi\Sigma$, VB means $\rho\Lambda$, $\rho\Sigma$ or K^*N and Y means Σ or Λ . The coupling constants are adopted as

$g_{KK\sigma}^2/4\pi = 0.25$ [33], $g_{K^*K\pi} = -3.23$, $f_{KN\Lambda} = 13.24$ and $f_{KN\Sigma} = 3.58$ [34]. The coupling constants about Σ^* can be obtained through $f_{\Sigma^*\Lambda\pi} = 1.27$, $f_{KN\Sigma^*} = -3.22$ [34] and $f_{\Delta\rho N} = -6.08$ [27] with the $SU(3)$ symmetry relations $f_{\pi\Sigma^*}/m_\pi = -\frac{1}{\sqrt{3}}\frac{f_{\pi\Lambda\Sigma^*}}{m_\pi}$, $\frac{f_{\rho\Lambda\Sigma^*}}{m_\rho} = \frac{1}{\sqrt{2}}f_{\Delta\rho N}/m_\rho$, $\frac{f_{\rho\Sigma^*}}{m_\rho} = -\frac{1}{\sqrt{6}}f_{\Delta\rho N}/m_\rho$ and $\frac{f_{K^*N\Sigma^*}}{m_{K^*}} = -\frac{1}{\sqrt{6}}f_{\Delta\rho N}/m_\rho$.

The Σ^* carries spin-parity $J^P = 3/2^+$ and isospin $I = 1$. A system composed of Σ^* and kaon carries $I = 1/2$ or $3/2$. In this work, all states with $J \leq 3/2$ are considered and the ranges of the cutoffs in form factors are chosen as $1 < \Lambda < 5$ GeV. The bound state solutions with the binding energies $E = m_1 + m_2 - W$ are listed in Table II and compared with the values from the PDG and the BnGa groups [1, 2].

TABLE II: The binding energies E for Σ^*K system with different cut off Λ . The cut off Λ , binding energy and branch ratio are in the units of GeV, MeV and %, respectively.

Λ	E	Γ	$N\sigma$	$N\rho$	$N\omega$	$N\pi$	ΛK	ΣK
1.68	3	41	55.9	4.7	14.1	22.4	2.3	0.6
1.72	8	73	55.8	4.7	14.0	22.6	2.3	0.6
1.76	16	111	55.7	4.7	14.0	22.7	2.2	0.6
1.80	28	155	55.6	4.8	14.2	22.8	2.1	0.5
1.84	44	204	55.3	4.9	14.6	22.7	2.0	0.5
1.88	67	257	54.9	5.1	14.9	22.9	1.8	0.4
1.92	100	312	53.6	5.1	14.7	24.8	1.5	0.3
PDG [1]	30^{+25}_{-25}		24^{+24}_{-24}	6^{+6}_{-6}	20^{+4}_{-4}	7^{+6}_{-6}		$0.7^{+0.4}_{-0.4}$
BnGa [2]	0^{+20}_{-20}	200^{+20}_{-20}	60^{+12}_{-12}			3^{+2}_{-2}	4^{+2}_{-2}	15^{+8}_{-8}
$[N(\frac{3}{2}^-)]_3$	-85	324		57.1	12.3	20.8	9.7	0

Only one bound state solution with $I = 1/2$ and $J^P = 3/2^-$ is found from the interaction of Σ^* and K . The decay width becomes larger with increase of the binding energy. It is understandable because the large binding energy means that the distance between two constituents is smaller so that the quark ex-

change is prone to happen in the bound state. Compared with the PDG and BnGa values about mass and total width, the best cut off $\Lambda \approx 1.80$ GeV is reasonable and consistent to the value in the literature [35]. The branch ratios of $N(1875)$ are stable compared with binding energy. The $N\sigma$ channel is the most important decay channel, about 55%, which is consistent with the PDG suggested values $24 \pm 24\%$ and $60 \pm 12\%$ from the BnGa analysis. The main decay channel of the $[N3/2^-]_3$ predicted in the constituent quark model is $N\rho$ which is much larger than other decay channels. It conflict with both the values suggested by the PDG and these obtained by the BnGa analysis. Hence, the decay pattern of $N(1875)$ disfavors the assignment as $[N3/2^-]_3$.

IV. SUMMARY

In this work, the internal structures of the (1875) and the $N(2120)$ are investigated. The experimental data for the photoproduction of $\Lambda(1520)$ off proton released by the CLAS and LEPS Collaborations suggest the explanation of the $N(2120)$ as the third state with $J^P = 3/2^-$ in the constituent quark model. The $N(1875)$ is explained as a bound state from the interaction of Σ^* and kaon, which is supported by the numerical results of both binding energy and decay pattern of the bound state of Σ^*K system with isospin $I = 1/2$ and spin-parity $J^P = 3/2^-$.

Acknowledgments

This project is partially supported by the Major State Basic Research Development Program in China (No. 2014CB845405), the National Natural Science Foundation of China (Grants No. 11275235, No. 11035006) and the Chinese Academy of Sciences (the Knowledge Innovation Project under Grant No. KJCX2-EW-N01).

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